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Construction of local antimagic 3-colorable graphs of fixed even size: Matrix approach

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Abstract. An edge labeling of a connected graph $G = (V, E)$ containing m edges is said to be local antimagic if it is a bijection $f : E \to \{1, \ldots, m\}$ such that, for any pair of adjacent vertices u and v, we have $f^+(u) \neq f^+(v)$, where the induced vertex label $f^+(u) = \sum f(e)$ with e ranging over all the edges incident with u. The local antimagic chromatic number of G, denoted by $\chi_{la}(G)$, is the minimum number of distinct induced vertex labels over all local antimagic labelings of G. In this paper, we give ways to construct matrices with integers in $[1, 10k]$, $k \geq 1$, that meet certain properties. Consequently, we obtain many families of (disconnected) bipartite (and tripartite) graphs of size $10k$ with local antimagic chromatic number 3.

1 Introduction

Let $G = (V, E)$ be a connected graph of order n and size m. A bijection $f : E \to \{1, 2, \ldots, m\}$ is called a *local antimagic labeling* if $f^+(u) \neq f^+(v)$ whenever $uv \in E$, where $f^+(u) = \sum_{e \in E(u)} f(e)$ and $E(u)$ is the set of edges incident with u. The mapping f^+, which is also denoted by f^+_G, is called a *vertex labeling of G induced by f*, and the labels assigned to vertices are called *induced colors* under f. The *color number* of a local antimagic labeling f is the number of distinct induced colors under f, denoted by $c(f)$. Moreover, f is called a *local antimagic $c(f)$-coloring*, and G is local antimagic $c(f)$-colorable. The *local antimagic chromatic number* $\chi_{la}(G)$ is defined to be the minimum number of colors taken over all colorings of G induced by local antimagic labelings of G, see [1]. In [2], Haslegrave proved that each connected graph except K_2 admits a local antimagic labeling.

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Thus, local antimagic chromatic number is well defined for all graphs without a K_2 component. Let $G + H$ and mG denote the disjoint union of graphs G and H, and m copies of G, respectively. For integers $a < b$, let $[a, b] = \{a, a + 1, \dots, b\}$.

The use of matrices that satisfy various properties have been used to determine the local antimagic chromatic number of many standard graphs (see [3–6]). However, the matrix in use either has empty cells or repeated entries. It is interesting to obtain matrices without empty cells nor repeated entries such that the entries correspond to a local antimagic $\chi_{la}(G)$-coloring of a graph G. In [7], the authors used matrices of size $5 \times (2k + 1)$ and $11 \times (2k+1)$ to construct various families of (disconnected) tripartite graphs of odd size $5 \times (2k + 1)$ and $11 \times (2k + 1)$ with local antimagic chromatic number 3. In this paper, we give ways to construct various matrices with even number of entries that meet certain properties. Consequently, we obtain many new (disconnected) graphs (both bipartite and tripartite) of even size with local antimagic chromatic number 3.

We shall need the following lemma.

Lemma 1.1 ([6]). *Let G be a graph of size q. Suppose there is a local antimagic labeling of G inducing a 2-coloring of G with colors x and y, where $x < y$. Let X and Y be the sets of vertices colored x and y, respectively. Then G is a bipartite graph with bipartition (X, Y) and $|X| > |Y|$. Moreover,*

$$x|X| = y|Y| = \frac{q(q + 1)}{2}.$$

2 $5 \times 2k$ matrix

In this section, we refer to the following $5 \times 2k$ matrix (with entries in $[1, 10k]$ bijectively) to get our results.

i	1	2	3	$\cdots$	$k-1$	k	$k+1$	$k+2$	$\cdots$	$2k-2$	$2k-1$	$2k$
u_iw_i	1	$k+1$	$k+2$	$\cdots$	$2k-2$	$2k-1$	2	3	$\cdots$	$k-1$	k	$2k$
v_iw_i	$6k$	$6k+1$	$6k+2$	$\cdots$	$7k-2$	$7k-1$	$7k+1$	$7k+2$	$\cdots$	$8k-2$	$8k-1$	$8k$
x_iw_i	$7k$	$6k-1$	$6k-3$	$\cdots$	$4k+5$	$4k+3$	$6k-2$	$6k-4$	$\cdots$	$4k+4$	$4k+2$	$3k+1$
x_iu_i	$10k$	$9k$	$9k-1$	$\cdots$	$8k+3$	$8k+2$	$10k-1$	$10k-2$	$\cdots$	$9k+2$	$9k+1$	$8k+1$
x_iv_i	$4k+1$	$4k$	$4k-1$	$\cdots$	$3k+3$	$3k+2$	$3k$	$3k-1$	$\cdots$	$2k+3$	$2k+2$	$2k+1$

We have the following observations:

(1) In each column, the sum of the first 3 row entries is a constant $K_1 = 13k + 1$.

(2) In each column, the sum of the rows 1 and 4 (respectively, 2 and 5) entries is $K_2 = 10k + 1$.

(3) The sums of the last 3 row entries for columns 1 and $2k$ are $21k + 1$ and $13k + 3$ respectively, while the corresponding sum for columns 2 to k (respectively, $k + 1$ to $2k - 1$) form an arithmetic progression from $19k - 1$ to $15k + 7$ (respectively, from $19k - 3$ to $15k + 5$) in decrement of 4. Moreover, the corresponding sum for columns a and $2k + 1 - a$ is $34k + 4$ $(1 \leq a \leq k)$.

(4) The sum of all the last 3 row entries is $S = k(34k + 4)$.

(5) If $2k = rs$ with $r \geq 2$ and $s \geq 1$, then we can divide the table into r blocks of s column(s) with the j-th block containing $(j - 1)s + 1$, $(j - 1)s + 2$, $\ldots$, js columns. Suppose r is even. The sum of the row 3 entries in the j-th block, and the rows 4 and 5 entries in the $(r+1-j)$-th block $(1 \leq j \leq r/2)$ is $K_3 = s(17k+2)$. Similarly when r is odd and $1 \leq j \leq (r - 1)/2$, the sum of the last three row entries in the $(r + 1)/2$ block is also K_3.

(6) For $1 \leq i \leq 2k$, the sum of the rows 2 and 3 entries of column i and the row 4 entry of column $2k + 1 - i$ is a constant $K_4 = 21k + 1$.

(7) For $1 \leq i \leq k$, the sum of row 4 columns i and $2k + 1 - i$ entries is a constant $18k + 1$, while the sum of row 5 columns i and $2k + 1 - i$ entries is a constant $6k + 2$.

The join graph of the graphs G and H is denoted by $G \vee H$. We denote $nP_3 \vee K_1$ by $FB(n)$, the *fan graph with n blades*. Note that $FB(1) = P_3 \vee K_1 \cong K_{1,2} \vee K_1$ is also the fan graph F_3 of order 4. In [7, Theorem 2.1], the authors have proved that $\chi_{la}(FB(n)) = 3$ for odd $n \geq 1$. We now extend it to even $n \geq 2$.

Theorem 2.1. *For $k \geq 1$, we have $\chi_{la}(FB(2k)) = 3$.*

Proof. Note that $FB(2k)$ is of size $10k$. Let the vertex set and edge set of the i-th copy of $FB(1)$ be $\{u_i, v_i, w_i, x_i\}$ and $\{u_iw_i, v_iw_i, x_iw_i, x_iu_i, x_iv_i\}$, respectively, $1 \leq i \leq 2k$. Observe that the table above gives a bijective edge labeling f of $2k$ copies of $FB(1)$ using integers in $[1, 10k]$ with induced vertex labels

$$f^+(u_i) = f^+(v_i) = 10k + 1, \qquad f^+(w_i) = 13k + 1,$$
$$\sum_{i=1}^{2k} f^+(x_i) = k(34k + 4),$$

for $1 \le i \le 2k$. Merging the vertices x_1 to x_{2k} gives us $FB(2k)$ that has a vertex x with $f^+(x) = k(34k + 4)$. Thus, $\chi_{la}(FB(2k)) \le 3$. Since $\chi_{la}(FB(2k)) \ge \chi(FB(2k)) = 3$, the theorem holds. □

Example 2.2. With $k = 6$ and the following table

i	1	2	3	4	5	6	7	8	9	10	11	12
u_iw_i	1	7	8	9	10	11	2	3	4	5	6	12
v_iw_i	36	37	38	39	40	41	43	44	45	46	47	48
x_iw_i	42	35	33	31	29	27	34	32	30	28	26	19
x_iu_i	60	54	53	52	51	50	59	58	57	56	55	49
x_iv_i	25	24	23	22	21	20	18	17	16	15	14	13

we have a local antimagic labeling for $12FB(1)$ as shown in Figure 2.1.

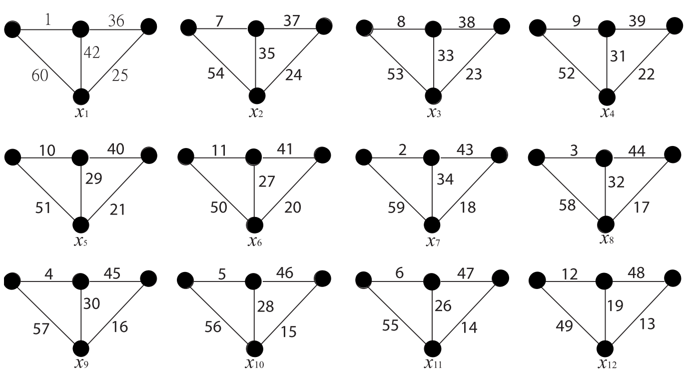

Figure 2.1: Twelve graphs that correspond to the 12 columns above.

Merging vertices x_1 to x_{12} gives the vertex x of $FB(12)$ and the corresponding local antimagic 3-coloring.

Theorem 2.3. *If $rs \ge 4$ with $r \ge 2$ and even $s \ge 2$, then $\chi_{la}(rFB(s)) = 3$.*

Proof. Begin with $2k = rs$ copies of $FB(1)$ with edge labeling as in the proof of Theorem 2.1 and keep all the notation.

For each $j \in [1, r]$, we merge the vertices in

$$\big\{x_{(j-1)s/2+a}, x_{2k-(j-1)s/2+1-a} \;\big|\; 1 \le a \le s/2\big\}.$$

We immediately get r copies of $FB(s)$ with a bijective edge labeling using integers in $[1, 10k]$. By Observation (3) above, $rFB(s)$ admits a local antimagic 3-coloring with induced vertex labels $10k+1$, $13k+1$, and $s(17k+2)$. Thus, $\chi_{la}(r(FB(s))) \le 3$. Since $\chi_{la}(r(FB(s))) \ge \chi(r(FB(s))) = 3$, the theorem holds. $\square$

Example 2.4. Consider $n = 12$. We can obtain $3FB(4)$, $6FB(2)$, and $2FB(6)$ from $12FB(1)$ with corresponding edge labeling similar to that in Example 2.2 by merging the vertices, respectively, in

$$\{x_{2i-1}, x_{2i}, x_{13-2i}, x_{14-2i}\}, \text{ for } 1 \le i \le 3;$$
$$\{x_i, x_{13-i}\}, \text{ for } 1 \le i \le 6;$$
$$\{x_{3i-2}, x_{3i-1}, x_{3i}, x_{13-3i}, x_{14-3i}, x_{15-3i}\}, \text{ for } 1 \le i \le 2.$$

For $r, s \ge 2$, $1 \le i \le r$, let $G_i \cong FB(s)$ with vertex set

$$\{u_{i,j}, v_{i,j}, w_{i,j}, x_i \mid 1 \le j \le s\}$$

and edge set

$$\{u_{i,j}w_{i,j}, v_{i,j}w_{i,j}, x_iw_{i,j} \mid 1 \le j \le s\}.$$

Note that $u_{i,j}$ and $v_{i,j}$ are of degree 2, $w_{i,j}$ is of degree 3, and x_i is of degree $3s$.

Denote by $FB_1(r, s)$ the graph obtained from $G_1, \ldots, G_r$ by merging the vertices in $\{u_{i,j} \mid 1 \le i \le r\}$ and in $\{v_{i,j} \mid 1 \le i \le r\}$, respectively for each $1 \le j \le s$. Let the new vertices be u_j and v_j, respectively for $1 \le j \le s$. Note that $FB_1(r, s)$ has rs vertices of degree 3, $2s$ vertices of degree $2r$, and r vertices of degree $3s$.

Denote by $FB_2(r, s)$ the graph obtained from $G_1, \ldots, G_r$ by merging the vertices in $\{w_{i,j} \mid 1 \le i \le r\}$, respectively for $1 \le j \le s$. Let the new vertices be w_j for $1 \le j \le s$. Note that $FB_2(r, s)$ has $2rs$ vertices of degree 2, s vertices of degree $3r$, and r vertices of degree $3s$.

Theorem 2.5. *For $r \ge 2$ and even $s \ge 2$, we have $\chi_{la}(FB_1(r, s)) = 3$ if $r \not\equiv 0 \pmod 4$ and $\chi_{la}(FB_2(r, s)) = 3$ if $rs \not\equiv 0 \pmod 4$.*

Proof. Let $rs = 2k$ for even $s \geq 2$. Since $\chi_{la}(FB_l(r,s)) \geq \chi(FB_l(r,s)) = 3$ for $l = 1,2$, it suffices to show that $\chi_{la}(FB_l(r,s)) \leq 3$. Using the r copies of $FB(s)$ in Theorem 2.3 and the corresponding local antimagic 3-coloring by merging the degree 2 vertices as defined for $FB_1(r,s)$, we can immediately conclude that $FB_1(r,s)$ admits a local antimagic 3-coloring with each $w_{i,j}$ (respectively, u_j, v_j, and x_i) has induced vertex label $13k+1$ (respectively, $r(10k+1)$ and $s(17k+2)$), for $1 \leq j \leq s$ and $1 \leq i \leq r$.

Suppose $r(10k+1) = s(17k+2)$. If r is odd, then we have

$$r^2(10k+1) = rs(17k+2) = 2k(17k+2).$$

So, $2k+1 \equiv 2k^2 \pmod 4$, which is impossible. If r is even, then k is even. Since s is even, $r(10k+1) = s(17k+2)$ implies that $r \equiv 2s \equiv 0 \pmod 4$. Thus if $r \not\equiv 0 \pmod 4$, then $r(10k+1) \neq s(17k+2)$.

Similarly, by merging the degree 3 vertices as defined for $FB_2(r,s)$, we can immediately conclude that $FB_2(r,s)$ admits a local antimagic 3-coloring with each $u_{i,j}$, $v_{i,j}$ (respectively, w_j and x_i) has induced vertex label $10k+1$ (respectively, $r(13k+1)$ and $s(17k+2)$), for $1 \leq j \leq s$ and $1 \leq i \leq r$.

Suppose $r(13k+1) = s(17k+2)$. This implies that $2k(13k+1) = s^2(17k+2)$. Then $2k \equiv 0 \pmod 4$. Thus when $rs = 2k \not\equiv 0 \pmod 4$, then $r(13k+1) \neq s(17k+2)$. This completes the proof. $\square$

For $s \geq 1$, let S_1 and S_2 be two copies of sP_3. Let $DF(2s)$ be the *diamond fan graph* obtained from $S_1 + S_2$ by joining a vertex y (respectively, z) to every degree 1 vertex of S_1 (respectively, S_2) and every degree 2 vertex of S_2 (respectively, S_1). Thus, $DF(2s)$ has $4s$ vertices of degree 2, $2s$ vertices of degree 3, and 2 vertices of degree $3s$ with size $10s$.

For $r \geq 1$, let $DF_r(2s) = rDF(2s) + FB(s)$. Note that $DF_r(2s)$ is of size $5(2r+1)s$ and has the following:

1. $(4r+2)s$ vertices of degree 2; we denote these vertices by u_i and v_i, $1 \leq i \leq (2r+1)s$.
2. $(2r+1)s$ vertices of degree 3; we denote these vertices by w_i, $1 \leq i \leq (2r+1)s$;
3. $2r+1$ vertices of degree $3s$; we denote these vertices by x, y_i and z_i, $1 \leq i \leq r$. Hence s can be 1.

Theorem 2.6. *If $r, s \geq 1$, then $\chi_{la}(rDF(2s)) = 3$.*

Proof. Let $2rs = 2k \geq 4$. We first note that $rDF(2s)$ is a bipartite graph with equal partite set size. By Lemma 1.1, $\chi_{la}(rDF(2s)) \geq 3$. It suffices to show that $rDF(2s)$ admits a local antimagic 3-coloring.

Begin with $2rs = 2k$ copies of $FB(1)$ with edge labeling as in the proof of Theorem 2.1. Partition the $(2rs)FB(1)$ into $2r$ blocks of $s \geq 1$ copies of $FB(1)$ such that the j-th block has vertices $x_{(j-1)s+a}$ for $1 \leq j \leq 2r$ and $1 \leq a \leq s$. Split each x_i into x_i^1 and x_i^2 such that x_i^1 is adjacent to w_i and x_i^2 is adjacent to u_i, v_i. For each $1 \leq j \leq r$ and $1 \leq a \leq s$, merge the vertices in $\{x^1_{(j-1)s+a}, x^2_{(2r-j)s+a}\}$ to get vertex y_j and the vertices in $\{x^2_{(j-1)s+a}, x^1_{(2r-j)s+a}\}$ to get vertex z_j. We now have r copies of $DF(2s)$. By Observation (5), we conclude that $rDF(2s)$ admits a local antimagic 3-coloring with degree 2 (respectively, 3 and $3s$) vertices having induced vertex labels $10k+1$ (respectively, $13k+1$ and $s(17k+2)$). Thus, $\chi_{la}(rDF(2s)) \leq 3$. This completes the proof. □

Example 2.7. Using the twelve copies of $FB(1)$ as in Example 2.2 so that $k = 6$, we can take $r = 3$ and $s = 2$ to get $3DF(4)$ as shown in Figure 2.2.

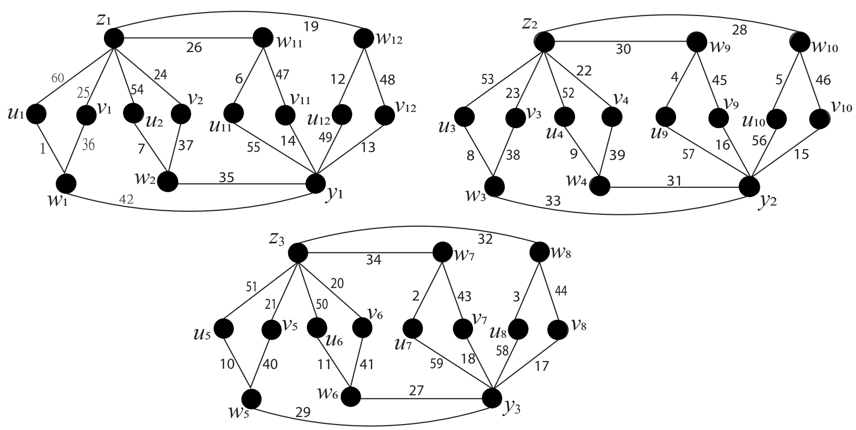

Figure 2.2: The $3DF(4)$ with the defined edge labeling.

Every degree 2 vertex has induced label 61, the degree 3 vertices have induced label 79, and the degree 6 vertex has induced label 208.

Theorem 2.8. *For $r \geq 1$ and even $s \geq 2$, we have $\chi_{la}(DF_r(2s)) = 3$.*

Proof. Let $(2r+1)s = 2k \geq 6$. Now, $DF_r(2s)$ has size $10k$. Since the $FB(s)$ component is a tripartite graph, we have $\chi_{la}(DF_r(2s)) \geq \chi(DF_r(2s)) = 3$. It suffices to show that $DF_r(2s)$ admits a local antimagic 3-coloring.

Begin with $(2r+1)s$ copies of $FB(1)$ with edge labeling as in the proof of Theorem 2.1. Partition the $(2r+1)sFB(1)$ into $2r+1$ blocks of even $s \geq 2$ copies of $FB(1)$ such that the j-th block has vertices $x_{(j-1)s+a}$ for $1 \leq j \leq 2r+1$ and $1 \leq a \leq s$. By the similar approach as in the proof of Theorem 2.6, we can construct the $rDF(2s)$ using the j-th blocks for $j \in [1, 2r+1] \setminus \{r+1\}$. The $(r+1)$-th block is used to construct the $FB(s)$ by merging vertices x_{rs+1} to $x_{(r+1)s}$. By Observation (5), we can also conclude that $DF_r(2s)$ that we obtained admits a local antimagic 3-coloring with degree 2 (respectively, 3 and $3s$) vertices having induced vertex labels $10k+1$ (respectively, $13k+1$ and $s(17k+2)$). This completes the proof. □

Example 2.9. Using the twelve copies of $FB(1)$ as in Example 2.2, we can take $r = 1$ and $s = 4$ to get $DF_1(8)$. Note that the $DF_1(8)$ can be obtained from the top two figures in Example 2.7 by merging y_1 and y_2, and also z_1 and z_2. The $FB(4)$ component can be obtained from Example 2.2 by merging vertices x_i, $i = 5, 6, 7, 8$.

For $1 \leq a \leq s$ and $1 \leq j \leq r$, consider the $rDF(2s)$ as in Theorem 2.6. Recall that the j-th component has

1. $4s$ vertices of degree 2, namely $u_{(j-1)s+a}$, $u_{(2r-j)s+a}$, $v_{(j-1)s+a}$, and $v_{(2r-j)s+a}$, $1 \leq a \leq s$;
2. $2s$ vertices of degree 3, namely $w_{(j-1)s+a}$ and $w_{(2r-j)s+a}$, $1 \leq a \leq s$; and
3. 2 vertices of degree $3s$, namely y_j and z_j.

We define the following graphs.

1. $DF^1(r, 2s)$ is the graph obtained from $rDF(2s)$ by merging the degree 3 vertices

$$\text{in } \{w_{(j-1)s+a} \mid 1 \leq j \leq r\} \text{ as } \alpha^{1,a},$$
$$\text{in } \{w_{(2r-j)s+a} \mid 1 \leq j \leq r\} \text{ as } \alpha^{2,a}.$$

   Note that the degree of $\alpha^{i,a}$ is $3r$, for $1 \leq i \leq 2$ and $1 \leq a \leq s$.

2. $DF^2(r, 2s)$ is the graph obtained from $rDF(2s)$ by merging the degree 2 vertices

$$\text{in } \{u_{(j-1)s+a} \mid 1 \le j \le r\} \text{ as } \beta^{1,a},$$
$$\text{in } \{u_{(2r-j)s+a} \mid 1 \le j \le r\} \text{ as } \beta^{2,a},$$
$$\text{in } \{v_{(j-1)s+a} \mid 1 \le j \le r\} \text{ as } \beta^{3,a},$$
$$\text{in } \{v_{(2r-j)s+a} \mid 1 \le j \le r\} \text{ as } \beta^{4,a}.$$

Note that the degree of $\beta^{i,a}$ is $2r$, for $1 \le i \le 4$ and $1 \le a \le s$.

3. $DF^3(r, 2s)$ is the graph obtained from $rDF(2s)$ by merging the degree $3s$ vertices

$$\text{in } \{y_j \mid 1 \le j \le r\} \text{ as } y,$$
$$\text{in } \{z_j \mid 1 \le j \le r\} \text{ as } z.$$

Note that the degree of y (and z) is $3rs$.

Further, for even $r \ge 2$, denote by $DF^4(r, 2s)$ the graph obtained from $rDF(2s)$ by merging y_j to z_{j+1} to obtain a degree $6s$ vertex, for $1 \le j \le r$, where $z_{r+1} = z_1$. Note that $DF^3(2, 2s) = DF^4(2, 2s)$.

Theorem 2.10. *For $r \ge 2$ and $s \ge 1$,*

1. $\chi_{la}(DF^1(r, 2s)) = 3$ *if s is even and* $rs \not\equiv 0 \pmod 4$*;*
2. $\chi_{la}(DF^2(r, 2s)) = 3$ *if s is even and* $r \not\equiv 0 \pmod 4$*;*
3. $\chi_{la}(DF^3(r, 2s)) = 3$*;*
4. $\chi_{la}(DF^4(r, 2s)) = 3$*.*

Proof. Note that $DF^i(r, 2s)$ is a bipartite graph with equal partite set size for each i, $1 \le i \le 4$. By Lemma 1.1, $\chi_{la}(DF^i(r, 2s)) \ge 3$. It suffices to show that $\chi_{la}(DF^i(r, 2s)) \le 3$ for $1 \le i \le 4$.

Let $k = rs$. Using the r copies of $DF(2s)$ in Theorem 2.6 and the corresponding local antimagic 3-coloring by merging the degree 3 vertices as defined for $DF^1(r, 2s)$, we can immediately conclude that $DF^1(r, 2s)$ admits a local antimagic 3-coloring having $2s$ (respectively, $4rs$ and $2r$) vertices of degree $3r$ (respectively, 2 and $3s$) with induced vertex label $r(13k + 1)$ (respectively, $10k + 1$ and $s(17k + 2)$). From the proof of Theorem 2.5, we have shown that $r(13k + 1) \ne s(17k + 2)$ under the assumption.

Merging the degree 2 vertices as defined for $DF^2(r, 2s)$, we can immediately conclude that $DF^2(r, 2s)$ admits a local antimagic 3-coloring having $4s$

(respectively, $2rs$ and $2r$) vertices of degree $2r$ (respectively, 3 and $3s$) with induced vertex label $r(10k+1)$ (respectively, $13k+1$ and $s(17k+2)$). From the proof of Theorem 2.5, we have shown that $r(10k+1) \neq s(17k+2)$ under the assumption.

Merging the degree $3s$ vertices as defined for $DF^3(r, 2s)$, we can immediately conclude that $DF^3(r, 2s)$ admits a local antimagic 3-coloring having $4rs$ (respectively, $2rs$ and 2) vertices of degree 2 (respectively, 3 and $3rs$) with induced vertex label $10k+1$ (respectively, $13k+1$, and $rs(17k+2)$).

Merging the degree $3s$ vertices pairwise as defined for $DF^4(r, 2s)$, we can immediately conclude that $DF^4(r, 2s)$ admits a local antimagic 3-coloring having r (respectively, $4rs$ and $2rs$) vertices of degree $6s$ (respectively, 2 and 3) with induced vertex label $2s(17k+2)$ (respectively, $10k+1$ and $13k+1$).

Thus, $\chi_{la}(DF^i(r, 2s)) \leq 3$ for $i = 1, 2, 3, 4$. □

3 $6 \times 4n$ matrix

We first construct a $6 \times 4n$ matrix, with $n \geq 1$, using integers in $[1, 20n]$ where integers in $[2n+1, 4n] \cup [16n+1, 18n]$ are used twice.

R_1	1	2	$\cdots$	$2n-1$	$2n$	$6n+2$	$6n+4$	$\cdots$	$10n-2$	$10n$	
R_2	$16n+1$	$16n+2$	$\cdots$	$18n-1$	$18n$	$18n$	$18n-1$	$\cdots$	$16n+2$	$16n+1$	$\leftarrow$
R_3	$14n-1$	$14n-3$	$\cdots$	$10n+3$	$10n+1$	$6n$	$6n-1$	$\cdots$	$4n+2$	$4n+1$	
R_4	$14n+1$	$14n+2$	$\cdots$	$16n-1$	$16n$	$6n+1$	$6n+3$	$\cdots$	$10n-3$	$10n-1$	
R_5	$2n+1$	$2n+2$	$\cdots$	$4n-1$	$4n$	$4n$	$4n-1$	$\cdots$	$2n+2$	$2n+1$	$\leftarrow$
R_6	$14n$	$14n-2$	$\cdots$	$10n+4$	$10n+2$	$20n$	$20n-1$	$\cdots$	$18n+2$	$18n+1$	

The "←" indicates numbers in that row appear twice. We are now ready to trace $2n$ sequences, denoted $T_1, T_2, \ldots, T_{2n}$, of length 12 as follows:

T_1	1	$16n+1$	$14n-1$	$6n+2$	$18n$	$6n$	$14n+1$	$2n+1$	$14n$	$6n+1$	$4n$	$20n$
T_2	2	$16n+2$	$14n-3$	$6n+4$	$18n-1$	$6n-1$	$14n+2$	$2n+2$	$14n-2$	$6n+3$	$4n-1$	$20n-1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
T_n	n	$17n$	$12n+1$	$8n$	$17n+1$	$5n+1$	$15n$	$3n$	$12n+2$	$8n-1$	$3n+1$	$19n+1$
T_{n+1}	$4n+1$	$16n+1$	$10n$	$10n+1$	$18n$	$2n$	$18n+1$	$2n+1$	$10n-1$	$10n+2$	$4n$	$16n$
T_{n+2}	$4n+2$	$16n+2$	$10n-2$	$10n+3$	$18n-1$	$2n-1$	$18n+2$	$2n+2$	$10n-3$	$10n+4$	$4n-1$	$16n-1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
T_{2n}	$5n$	$17n$	$8n+2$	$12n-1$	$17n+1$	$n+1$	$19n$	$3n$	$8n+1$	$12n$	$3n+1$	$15n+1$
common diff.	$+1$	$+1$ ↑	-2	$+2$	-1 ↑	-1	$+1$	$+1$ ↑	-2	$+2$	-1 ↑	-1

The "↑" indicates numbers in that column appear twice. It is easy to verify that there is a bijective mapping between the entries in the matrix above and the terms of the sequences.

Consider two 8-cycles $G = u_1u_2u_3 \cdots u_8u_1$ and $H = v_1v_2 \cdots v_8v_1$. The graph $C_4(8,2)$ is obtained from $G + H$ by adding the edges u_2v_2, u_4v_4, u_6v_6, and u_8v_8.

Theorem 3.1. *For $n \geq 1$, we have $\chi_{la}(nC_4(8,2)) = 3$.*

Proof. Let $G = nC_4(8,2)$. By definition, G is a bipartite graph with equal partite set size. By Lemma 1.1, $\chi_{la}(G) \geq 3$. It suffices to show that G admits a local antimagic 3-coloring.

We now have the following observations.

(1) For each T_a, $1 \leq a \leq 2n$, the sum of the first and last terms (respectively, the 3-rd and 4-th; the 6-th and 7-th; and the 9-th and 10-th terms) is a constant $20n + 1$.

(2) For each T_a, $1 \leq a \leq n$, the sum of the first 3 terms (respectively, the last 3 terms); whereas for $n+1 \leq a \leq 2n$, the sum of the 4-th to 6-th terms (respectively, the 7-th to 9-th terms), is a constant $30n + 1$.

(3) For each T_a, $1 \leq a \leq n$, the sum of the 4-th to 6-th terms (respectively, the 7-th to 9-th terms); whereas for $n + 1 \leq a \leq 2n$, the sum of the first 3 terms (respectively, the last 3 terms), is a constant $30n + 2$.

(4) For $1 \leq a \leq n$, T_a and T_{n+a} have the same second, fifth, eighth and eleventh terms.

For $1 \leq a \leq n$, let the vertex set and the edge set of the a-th copy of $C_4(8,2)$ be

$$\{u_{a,i}, v_{a,i} \mid 1 \leq i \leq 8\}$$

and

$$\{u_{a,i}u_{a,i+1}, v_{a,i}v_{a,i+1}, u_{a,2j}v_{a,2j} \mid 1 \leq i \leq 8, 1 \leq j \leq 4\},$$

respectively, where $u_{a,9} = u_{a,1}$, $v_{a,9} = v_{a,1}$. We now define a bijective function $f : E(G) \to [1, 20n]$ such that for the a-th copy of $C_4(8,2)$, f assigns the edges of the 8-cycle $u_{a,1}u_{a,2} \cdots u_{a,8}u_{a,1}$ (respectively, the 8-cycle $v_{a,1}v_{a,2} \cdots v_{a,8}v_{a,1}$) by the first, third, fourth, sixth, seventh, ninth, tenth and twelfth terms of T_a (respectively, T_{n+a}) consecutively, while the edges $u_{a,2}v_{a,2}$, $u_{a,4}v_{a,4}$, $u_{a,6}v_{a,6}$ and $u_{a,8}v_{a,8}$ are assigned by the second, fifth, eighth and eleventh terms of T_a. By the observations above, we can immediately conclude that f is a local antimagic labeling of G such that every degree 2 vertex has induced vertex label $20n + 1$, where every two adjacent degree 3 vertices have induced vertex $30n + 1$ and $30n + 2$ respectively. Thus, G admits a local antimagic 3-coloring. This completes the proof. □

Example 3.2. Taking $n = 6$, we have the following sequences:

T_1:	1,	97,	83,	38,	108,	36,	85,	13,	84,	37,	24,	120
T_2:	2,	98,	81,	40,	107,	35,	86,	14,	82,	39,	23,	119
T_3:	3,	99,	79,	42,	106,	34,	87,	15,	80,	41,	22,	118
T_4:	4,	100,	77,	44,	105,	33,	88,	16,	78,	43,	21,	117
T_5:	5,	101,	75,	46,	104,	32,	89,	17,	76,	45,	20,	116
T_6:	6,	102,	73,	48,	103,	31,	90,	18,	74,	47,	19,	115
T_7:	25,	97,	60,	61,	108,	12,	109,	13,	59,	62,	24,	96
T_8:	26,	98,	58,	63,	107,	11,	110,	14,	57,	64,	23,	95
T_9:	27,	99,	56,	65,	106,	10,	111,	15,	55,	66,	22,	94
T_{10}:	28,	100,	54,	67,	105,	9,	112,	16,	53,	68,	21,	93
T_{11}:	29,	101,	52,	69,	104,	8,	113,	17,	51,	70,	20,	92
T_{12}:	30,	102,	50,	71,	103,	7,	114,	18,	49,	72,	19,	91

Then $6C_4(8,2)$ is labeled as shown in Figure 3.1.

We keep the names of vertices of $nC_4(8,2)$ defined in the proof of Theorem 3.1. Suppose $n = rs$ with $r \ge 1$ and $s \ge 2$. Consider each $b \in [1,r]$. Let $G_1(r,s)$ be obtained from rs copies of $C_4(8,2)$ by merging degree 2 vertices in $\{u_{(b-1)s+i,2j-1} \mid 1 \le i \le s\}$ to get degree $2s$ vertices; and in $\{v_{(b-1)s+i,2j-1} \mid 1 \le i \le s\}$ to get degree $2s$ vertices, for each $j \in [1,4]$, respectively. Let $G_2(r,s)$ be obtained from rs copies of $C_4(8,2)$ by merging degree 3 vertices in $\{u_{(b-1)s+i,2j} \mid 1 \le i \le s\}$, for each $j = 1,4$ respectively, to get degree $3s$ vertices; and in $\{v_{(b-1)s+i,2j} \mid 1 \le i \le s\}$, for each $j = 2,3$ respectively, to get degree $3s$ vertices. Note that both $G_1(r,s)$ and $G_2(r,s)$ are bipartite graphs with equal partite set size having r component(s).

Theorem 3.3. *If $m = 1,2$, $r \ge 1$, and $s \ge 2$, then $\chi_{la}(G_m(r,s)) = 3$.*

Proof. For $m = 1,2$, since $G_m(r,s)$ is bipartite graph with equal partite set size, we have $\chi_{la}(G_m(r,s)) \ge 3$. It suffices to show that $G_m(r,s)$ admits a local antimagic 3-coloring. Let $n = rs$. Begin with the $nC_4(8,2)$ and the corresponding local antimagic 3-coloring as in the proof of Theorem 3.1. Note that $G_1(r,s)$ has vertices of degree $2s$ and 3. By way of construction of $G_1(r,s)$, we immediately have a local antimagic labeling with induced vertex label of degree $2s$ vertices is $s(20n+1)$ and every two adjacent degree 3 vertices still have induced vertex labels $30n+1$ and $30n+2$, respectively.

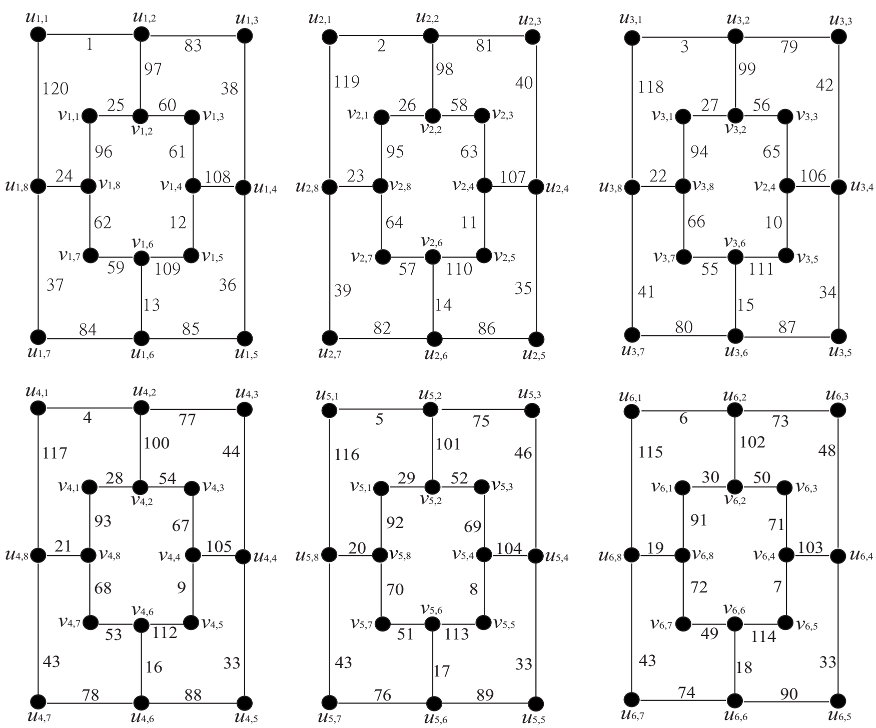

Figure 3.1: The $6C_4(8,2)$ with the defined edge labeling.

Note that $G_2(r,s)$ has vertices of degree 2, 3, and $3s$. Similarly, by way of construction of $G_2(r,s)$, we immediately have a local antimagic labeling with induced vertex label of degree 2 vertices is $20n+1$, of degree 3 vertices is $30n+2$, and of degree $3s$ vertices is $s(30n+1)$. Thus, $G_m(r,s)$, $m=1,2$, admits a local antimagic 3-coloring. This completes the proof. $\square$

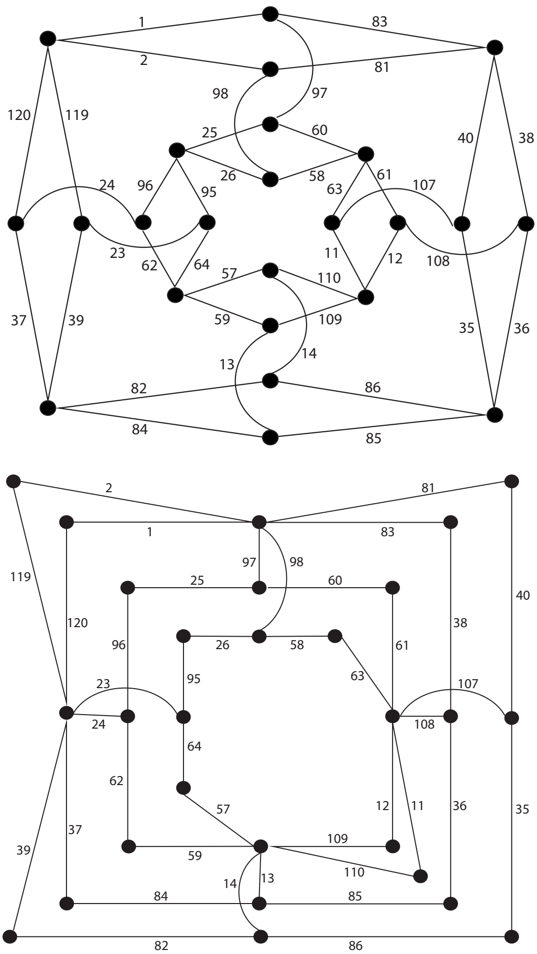

Figure 3.2: The first component of $G_m(3,2)$, $m=1,2$, with the defined edge labeling.

Example 3.4. Using the first two components of $6C_4(8,2)$, we now give, in Figure 3.2, a component of the graphs $G_m(3,2)$ for $m = 1,2$. Two other components can be obtained similarly.

(1) Let $H_1(n)$ be obtained from n copies of $C_4(8,2)$ by merging degree 2 vertices in $\{u_{i,1}, u_{i,5}\}$ to get degree 4 vertex $x_{i,1}$; in $\{u_{i,3}, u_{i,7}\}$ to get degree 4 vertex $x_{i,2}$; in $\{v_{i,1}, v_{i,5}\}$ to get degree 4 vertex $y_{i,1}$; and in $\{v_{i,3}, v_{i,7}\}$ to get degree 4 vertex $y_{i,2}$, for each $i \in [1,n]$.

(2) Let $H_2(n)$ be obtained from n copies of $C_4(8,2)$ by merging the vertices in $\{u_{i,1}, v_{i,7}\}$ to get degree 4 vertex $x_{i,1}$; in $\{u_{i,5}, v_{i,3}\}$ to get degree 4 vertex $x_{i,2}$; in $\{u_{i,3}, v_{i,5}\}$ to get degree 4 vertex $y_{i,1}$; in $\{u_{i,7}, v_{i,1}\}$ to get degree 4 vertex $y_{i,2}$, for each $i \in [1,n]$.

(3) Let $H_3(n)$ be obtained from n copies of $C_4(8,2)$ by merging the vertices in $\{u_{i,1}, v_{i,1}\}$ to get degree 4 vertex $x_{i,1}$; in $\{u_{i,5}, v_{i,5}\}$ to get degree 4 vertex $x_{i,2}$; in $\{u_{i,3}, v_{i,3}\}$ to get degree 4 vertex $y_{i,1}$; in $\{u_{i,7}, v_{i,7}\}$ to get degree 4 vertex $y_{i,2}$, for each $i \in [1,n]$.

Note that $H_1(n)$ is a bipartite graph with equal partite set size while $H_2(n)$ and $H_3(n)$ are tripartite graphs.

Theorem 3.5. *If $n \geq 1$ and $m = 1,2,3$, then $\chi_{la}(H_m(n)) = 3$.*

Proof. Since $H_1(n)$ is a bipartite graph with equal partite set size, we have $\chi_{la}(H_1(n)) \geq 3$. It suffices to show that $H_1(n)$ admits a local antimagic 3-coloring. Begin with the $nC_4(8,2)$ and the corresponding local antimagic 3-coloring as in the proof of Theorem 3.1. By way of construction of $H_1(n)$, we immediately have a local antimagic labeling with induced vertex label of every two adjacent degree 3 vertices are $30n+1$ and $30n+2$, respectively, and of the degree 4 vertices is $40n+2$.

Since $H_2(n)$ is tripartite, $\chi_{la}(H_2(n)) \geq 3$. Similarly, by way of construction of $H_2(n)$, we immediately have a local antimagic labeling with induced vertex label of every degree 4 vertex is $40n+2$, and every two adjacent degree 3 vertices are $30n+1$ and $30n+2$, respectively. Thus, $H_m(n)$ $(m = 1,2)$ admits a local antimagic 3-coloring.

Since $H_3(n)$ is tripartite, $\chi_{la}(H_3(n)) \geq 3$. Similarly, by way of construction of $H_3(n)$, we immediately have a local antimagic labeling with induced vertex label of every degree 4 vertex is $40n+2$, and every two adjacent degree 3 vertices are $30n+1$ and $30n+2$, respectively. Thus, $H_m(n)$ $(m = 1,2,3)$ admits a local antimagic 3-coloring. This completes the proof. □

Example 3.6. Using the first component of $6C_4(8,2)$, we now give the first component of the graphs $H_m(6)$ for $m = 1,2,3$ as shown in Figure 3.3. Five other components can be obtained similarly.

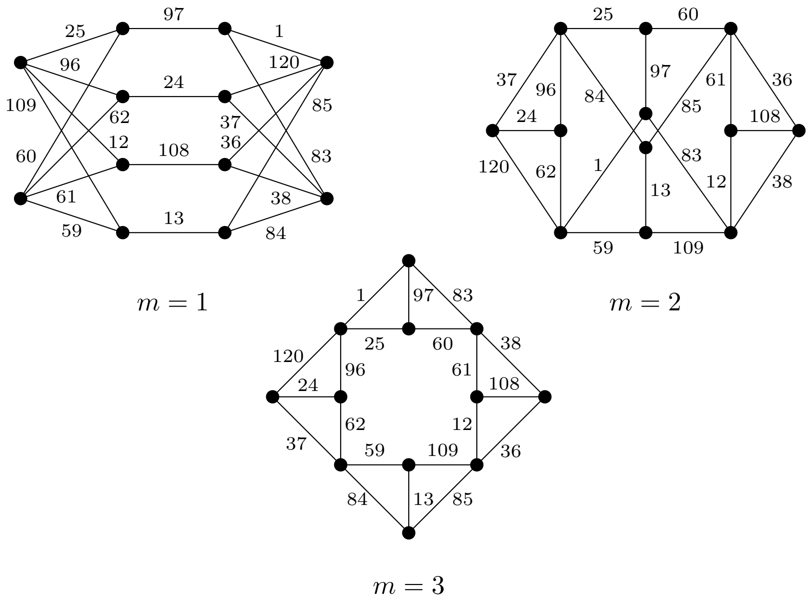

Figure 3.3: The first component of $H_m(6)$, $m = 1,2,3$ with the defined edge labeling.

Suppose $n = rs$ with $r \geq 1$ and $s \geq 2$. For $m = 1,2,3$ and each $b \in [1,r]$ and $j = 1,2$, let $H_m(r,s)$ be the graph obtained from $H_m(r,s)$ by merging degree 4 vertices in $\{x_{(b-1)s+i,j} \mid i \in [1,s]\}$ to get degree $4s$ vertices; and in $\{y_{(b-1)s+i,j} \mid i \in [1,s]\}$ to get degree $4s$ vertices. Note that $H_1(r,s)$ is a bipartite graph with each component having equal partite set size so that $\chi_{la}(H_1(r,s)) \geq 3$. Moreover, $\chi_{la}(H_m(r,s)) \geq \chi(H_m(r,s)) = 3$ for $m = 2,3$. By using the same local antimagic 3-coloring of $H_m(n)$, we get a new labeling for $H_m(r,s)$ with distinct induced vertex labels $s(40n+2)$, $30n+1$, and $30n+2$. So we have the following result.

Theorem 3.7. *If $r \geq 1$, $s \geq 2$, and $m = 1,2,3$, then $\chi_{la}(H_m(r,s)) = 3$.*

4 $k \times 10$ matrix

We now construct a $k \times 10$ matrix for $k \geq 1$ (also with integers in $[1, 10k]$) as follows to obtain all our remaining results.

R_1	1	$6k$	$4k+1$	$2k+1$	$8k$	$2k$	$8k+1$	$10k$	$7k$	$3k+1$
R_2	$k+1$	$6k+1$	$4k$	$2k+2$	$8k-1$	k	$9k+1$	$9k$	$6k-1$	$4k+2$
R_3	$k+2$	$6k+2$	$4k-1$	$2k+3$	$8k-2$	$k-1$	$9k+2$	$9k-1$	$6k-3$	$4k+4$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$
R_{k-1}	$2k-2$	$7k-2$	$3k+3$	$3k-1$	$7k+2$	3	$10k-2$	$8k+3$	$4k+5$	$6k-4$
R_k	$2k-1$	$7k-1$	$3k+2$	$3k$	$7k+1$	2	$10k-1$	$8k+2$	$4k+3$	$6k-2$

We now have the following observations:

(a) In each row, the sum of the entries in columns $1, 8$ (respectively, $2, 3$; $4, 5$; $6, 7$; and $9, 10$) is the constant $10k + 1$.

(b) In each row, the sum of the entries in columns $1, 2, 9$ (respectively, $5, 6, 10$) is the constant $13k + 1$.

Let $C_8(P_3) = C_8 \cup P_3$, where $C_8 = u_1u_2u_3u_4u_5u_6u_7u_8u_1$ and $P_3 = u_2xu_6$. Now we consider $kC_8(P_3)$ for $k \geq 1$. For the i-th copy of $C_8(P_3)$, we rewrite the vertices u_j and x by $u_{i,j}$ and x_i, respectively, where $1 \leq j \leq 8$ and $1 \leq i \leq k$. Now the vertex $u_{i,j}$, $j = 1, 3, 5, 7$ (respectively, $j = 2, 6$) are of degree 2 (respectively, 3) with induced vertex label $10k+1$ (respectively, $13k + 2$). Moreover, vertices x_i, $u_{i,4}$, and $u_{i,8}$ are of degree 2 with induced vertex label $10k + 1$, $6k + 2$, and $18k + 1$, respectively. Thus we have the following result.

Theorem 4.1. *If $k \geq 1$, then $2 \leq \chi_{la}(kC_8(P_3)) \leq 4$.*

Example 4.2. Take $k = 4$. We get the following table and the corresponding labeling for $4C_8(P_3)$ as shown in Figure 4.1.

R_1	1	24	17	9	32	8	33	40	28	13
R_2	5	25	16	10	31	4	37	36	23	18
R_3	6	26	15	11	30	3	38	35	21	20
R_4	7	27	14	12	29	2	39	34	19	22

Let B_8 be the bipartite graph obtained from $C_8(P_3)$ by merging the vertices u_4 and u_8.

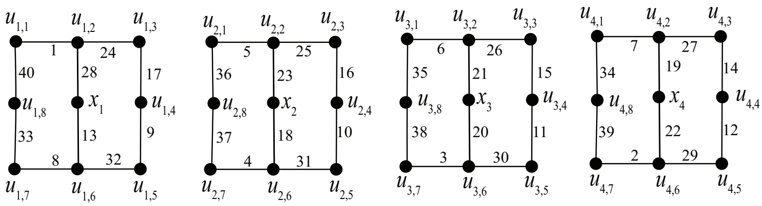

Figure 4.1: Graph $4C_8(P_3)$ with the defined edge labeling.

Theorem 4.3. *If $k \geq 1$, then $2 \leq kB_8 \leq 3$.*

Proof. Keeping the labeling as defined under Theorem 4.1, we conclude that kB_8 admits a local antimagic 3-coloring with induced vertex labels $10k+1$, $13k+2$, and $24k+3$. This completes the proof. □

Let A_8 be the graph obtained from $C_8(P_3)$ by merging vertices x and u_8. Let the new vertex be z. Now we consider kA_8 for $k \geq 1$. For the i-th copy of A_8, which we denote by A_8^i, we rewrite the vertices u_j and z by $u_{i,j}$ and z_i, respectively, where $1 \leq j \leq 8$ and $1 \leq i \leq k$. Thus, vertex z_i is of degree 4 with induced vertex label $28k+2$.

Theorem 4.4. *If $k \geq 1$, then $3 \leq \chi_{la}(kA_8) \leq 4$.*

Proof. From the observations above, we can immediately conclude that kA_8 admits a local antimagic 4-coloring with each vertex $u_{i,j}, j = 1,3,5,7$ of degree 2 (respectively, $u_{i,4}$ of degree 2; $u_{i,j}, j = 2,6$ of degree 3; and z_i of degree 4) has induced vertex label $10k+1$ (respectively, $6k+2$, $13k+1$, and $28k+2$). Thus, $\chi_{la}(kA_8) \leq 4$. Since $\chi_{la}(kA_8) \geq \chi(kA_8) = 3$, the theorem holds. □

Let D_8 be the graph obtained from $C_8(P_3)$ by merging vertices x, u_4, and u_8. Let the new vertex be w. Now we consider kD_8 for $k \geq 1$. For the i-th copy of D_8, we rewrite the vertices u_j and w by $u_{i,j}$ and w_i, respectively, where $1 \leq j \leq 8$ and $1 \leq i \leq k$.

Theorem 4.5. *If $k \geq 1$, then $\chi_{la}(kD_8) = 3$.*

Proof. From the observations above, we can immediately conclude that kD_8 admits a local antimagic 3-coloring. Since $\chi(kD_8) = 3$, the theorem holds. □

Let $k = rs \geq 2$. Suppose s is an even positive integer. For $1 \leq i \leq s/2$ and $1 \leq a \leq r$, let $G(8, s)$ be obtained from s copies of A_8 by merging vertices in

$$\{z_{(a-1)s+i}, u_{(2a-1)s/2+i,4}\}$$

and in

$$\{z_{(2a-1)s/2+i}, u_{(a-1)s+i,4}\},$$

respectively.

Theorem 4.6. *For $r \geq 1$ and even $s \geq 2$, we have $\chi_{la}(rG(8, s)) = 3$.*

Proof. From the construction of $G(8, s)$, we know

$$\chi_{la}(rG(8, s)) \geq \chi(rG(8, s)) = 3.$$

It suffices to show that $rG(8, s)$ admits a local antimagic 3-coloring. Observe that the edge labeling defined for the i-th copy of $C_8(P_3)$ $(1 \leq i \leq k)$ corresponds to a local antimagic labeling of $rG(8, s)$ such that all the $4k$ vertices of degree 2 have induced vertex label $10k + 1$, all the $2k$ vertices of degree 3 have induced vertex label $13k + 1$, and all the $2r$ vertices of degree $3s$ have induced vertex label $s(17k + 2)$. Thus, $\chi_{la}(rG(8, s)) \leq 3$. This completes the proof. □

Example 4.7. From the $4A_8$, we can take $r = 1$ so that $G(8, 4)$ is connected, or we can take $r = s = 2$ so that $2G(8, 2)$ has 2 components. The graph of the latter case is shown in Figure 4.2.

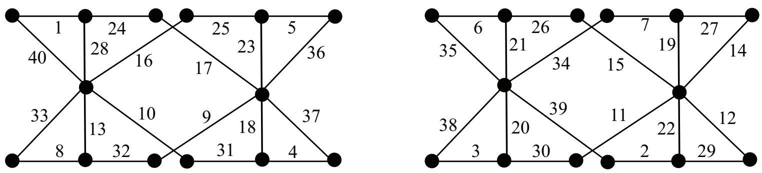

Figure 4.2: Graph $2G(8, 2)$ with the defined edge labeling.

Suppose $k = rs$ is odd. Let $H(r,s)$ be obtained from s copies of A_8^i by merging vertices in $\{z_i, u_{(k+1)/2+i}\}$ and in $\{z_{(k+1)/2+j}, u_{j,4}\}$ for $1 \le i \le (k+1)/2$ and $1 \le j \le (k-1)/2$.

Theorem 4.8. *If $rs \ge 3$ and rs is odd, then $3 \le \chi_{la}(H(r,s)) \le 4$.*

Proof. Clearly, $\chi_{la}(H(r,s)) \ge \chi(H(r,s)) = 3$. It suffices to show that

$$\chi_{la}(H(r,s)) \le 4$$

for $k \ge 3$. From the construction of $H(r,s)$, we observe the following:

- The edge labeling defined for the i-th copy of $C_8(P_3)$ $(1 \le i \le k)$ corresponds to a local antimagic labeling of H such that all $4k$ vertices of degree 2 have induced vertex label $10k+1$.
- All the $2k$ vertices of degree 3 have induced vertex label $13k+2$.
- There are r vertices of degree $3(s-1)+4$ with induced vertex label $(s-1)(17k+2)+18k+1$ and another r vertices of degree $3(s-1)+2$ with induced vertex label $(s-1)(17k+2)+6k+2$.

Thus, $\chi_{la}(H) \le 4$. □

5 Conclusion and open problems

In this paper, we have constructed three matrices of size $5 \times 2k$, $6 \times 2k$ (k even), and $k \times 10$, respectively with entries in $[1, 10k]$ that satisfy certain properties. Consequently, many bipartite and tripartite graphs with local antimagic chromatic number 3 are obtained. We have proved that $\chi_{la}(rFB(s)) = 3$ for odd $r, s \ge 3$ in [7, Theorem 2.2].

Conjecture 5.1. For even $r \ge 2$ and odd $s \ge 3$, we expect $\chi_{la}(rFB(s)) = 3$.

Note that each of the n components of $H_3(n)$ in Theorem 3.5 is the triangular bracelet $TB(3)$ defined in [7, Theorem 3.2]. Thus, we necessarily have $\chi_{la}(k(TB(3))) = 3$ for $k \ge 1$. We end this paper with the following problems that arise naturally.

Problem 5.2. For odd s or $rs \equiv 0 \pmod 4$, determine $\chi_{la}(DF^i(r,2s))$, $i = 1,2$.

Problem 5.3. Determine $\chi_{la}(k(TB(n))$ for $k \ge 2$ and $n \ge 1$.

Problem 5.4. For $k \geq 1$, determine $\chi_{la}(kB_8)$.

Problem 5.5. For $k \geq 1$, determine $\chi_{la}(kC_8(P_3))$.

Problem 5.6. For $k \geq 1$, determine $\chi_{la}(kA_8)$.

Problem 5.7. For $rs \geq 3$ and rs is odd, determine $\chi_{la}(H(r,s))$.

The Cartesian product of two graphs G and H is denoted by $G \times H$. Now, consider the plane graph $C_p \times P_2$, where $p \geq 2$. We now define graph(s) $C_q(p,2)$ obtained from $C_p \times P_2$ by deleting q edges not in C_p and each pair of these q edges are not incident with the same face, where $1 \leq q \leq \frac{p}{2}$. Note that $C_q(p,2)$ is unique up to isomorphism if and only if $p = 2q$ or $p = 2q+1$ or $q = 1$.

Problem 5.8. For $(p,q) \neq (8,4)$, determine $\chi_{la}(C_q(p,2))$.

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